9.1 ADDING AND SUBTRACTING RADICALS

RULE 1: To add or subtract you have to have the same name. In Algebra we call them like terms.
(Remember , every little piece of the names must be identical.)

EXAMPLE:

1. $3\sqrt{3} + 5\sqrt{3} = 8\sqrt{3}$

2. $3\sqrt{3} + 5\sqrt{7} =$ simplest form

3. $9\sqrt{6} - 4\sqrt{6} = 5\sqrt{6}$

4. $8\sqrt{11} - 3\sqrt{5} =$ simplest form

5. $7XY^2\sqrt{5} + 4XY^2\sqrt{5} = 11XY^2\sqrt{5}$

6. $9XY^2\sqrt{5} + 3XY\sqrt{5} =$ simplest form

PRACTICE

1. $5\sqrt{7} + 8\sqrt{7} =$ $13\sqrt{7}$

2. $11\sqrt{13} - 5\sqrt{13} =$ $6\sqrt{13}$

3. $7\sqrt{5} + 3\sqrt{5} =$ $10\sqrt{5}$

4. $13MN^3\sqrt{17} + 4MN^3\sqrt{17} =$ $17MN^3\sqrt{17}$

5. $19XH\sqrt{5} - 11XH^2\sqrt{5} =$ simplest form

6. $7CH\sqrt{3} + 3CH\sqrt{3} =$ $10CH\sqrt{3}$

7. $3KP\sqrt{11} + 7K^3P^2\sqrt{11} + 5KP\sqrt{11} + 8K^3P^2\sqrt{11} =$

$8KP\sqrt{11} + 15K^3P^2\sqrt{11}$

RULE 2: To multiply or divide you do NOT have to have the same name. They do NOT have to be like terms. Every time you multiply or divide you get a new name. (Don't forget exponent , distributive , and sign rules.)

EXAMPLE:

1. $4\sqrt{3} \times 5\sqrt{7} = 20\sqrt{21}$ 2. $16\sqrt{15} \div 8\sqrt{3} = 2\sqrt{5}$

3. $7\sqrt{11} \times 4\sqrt{5} = 28\sqrt{55}$ 4. $-24\sqrt{21} \div -6\sqrt{7} = 4\sqrt{3}$

5. $-5X^3M^5\sqrt{3} \times 7X^2M^2Q\sqrt{11} = -35X^5M^7Q\sqrt{33}$ 6. $\frac{28TR^5\sqrt{21}}{49T^3R^2\sqrt{7}} = \frac{4R^3\sqrt{3}}{7T^2}$

PRACTICE:

1. $-8\sqrt{13} \times -7\sqrt{5} = 56\sqrt{65}$ 2. $6\sqrt{35} \div 3\sqrt{5} = 2\sqrt{7}$

3. $-7FH\sqrt{5} \times 9FH^3\sqrt{3} = -63F^2H^4\sqrt{15}$ 4. $3Q\sqrt{13} \times 7D\sqrt{11} = 21QD\sqrt{143}$

5. $18X^5B^7\sqrt{55} \div 2X^3B\sqrt{5} = 9X^2B^6\sqrt{11}$

6. $5XY\sqrt{7}(-3X^2Y\sqrt{3} + -4X^3Y^3\sqrt{5}) + 8X^3Y^2\sqrt{21} = -7X^3y^2\sqrt{21} - 20x^4y^4\sqrt{35}$

RULE 1: To simplify a radical , you should ask yourself these four questions , in this order , after every single step.

1. Will anything reduce? (#'s with #'s $\sqrt{\ }$ with $\sqrt{\ }$)
2. Will a radical break down? (have sets of primes)
3. Is there a fraction in the $\sqrt{\ }$?
4. Is there a $\sqrt{\ }$ in the denominator?

When you get a NO to all four questions then it must be in simplest form.

FIXING PROBLEM 1 (REDUCING)

$\frac{28\sqrt{48}}{35\sqrt{10}} \div 7\sqrt{2} = \frac{4\sqrt{24}}{5\sqrt{5}}$ (still has problems)

$\frac{48\sqrt{35}}{32\sqrt{55}} \div 16\sqrt{5} = \frac{3\sqrt{7}}{2\sqrt{11}}$ (still has problems)

FIXING PROBLEM 2 (BREAKING DOWN RADICALS)

1. Do the prime factorization? (Primes -- 2,3,5,7,11,13,17,19, ect.)
2. Circle sets of 2 for $\sqrt{\ }$, 3 for $\sqrt[3]{\ }$, ect.
3. Take one from each set outside.
4. Leave those not in a set inside.
5. Multiply the outsides. Multiply the insides.

$\sqrt{4720}$

$\sqrt{(2\cdot2)(2\cdot2)\cdot5\cdot59}$

$2\cdot2\sqrt{5\cdot59}$

$4\sqrt{295}$

2 | 4720
2 | 2360
2 | 1180
2 | 590
5 | 295
59

$\sqrt[3]{4720}$

$\sqrt[3]{(2\cdot2\cdot2)\cdot2\cdot5\cdot59}$

$2\sqrt[3]{2\cdot5\cdot59}$

$2\sqrt[3]{590}$

$\sqrt{70}$

simplest form (no sets)

2 | 70
5 | 35
7

3. $\sqrt{48}$

$2|48$
$2|24$
$2|12$
$2|6$
3

$\sqrt{2\cdot2\cdot2\cdot2\cdot3}$

$2\cdot2\sqrt{3}$

$4\sqrt{3}$

4. $\dfrac{\sqrt{24}\div\sqrt{6}}{\sqrt{6}\div\sqrt{6}}$

$\sqrt{4} = 2$ or -2

5. $\dfrac{2}{\sqrt{2}}\cdot\dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{2\sqrt{2}}{2}$

$\sqrt{2}$

250

6. $\dfrac{42\sqrt[3]{50}\div6\sqrt[3]{10}}{36\sqrt[3]{30}\div6\sqrt[3]{10}} = \dfrac{7\sqrt[3]{5}\cdot\sqrt[3]{3\cdot3}}{6\sqrt[3]{3}\cdot\sqrt[3]{3\cdot3}} = \dfrac{7\sqrt[3]{45}}{18}$

$6\cdot3$

7. $\dfrac{(3+\sqrt{5})(2-\sqrt{7})}{(2+\sqrt{7})(2-\sqrt{7})} = \dfrac{6+2\sqrt{5}-3\sqrt{7}-\sqrt{35}}{4-7} = \dfrac{6+2\sqrt{5}-3\sqrt{7}-\sqrt{35}}{-3}$

we multiplied by the binomial conjugate

difference between two squares

$2^2-(\sqrt{7})^2$

$2|60$
$2|30$
$3|15$
5

8. $\dfrac{3\sqrt[3]{7}\cdot\sqrt[3]{2\cdot3\cdot3\cdot5\cdot5}}{5\sqrt[3]{60}\cdot\sqrt[3]{2\cdot3\cdot3\cdot5\cdot5}} = \dfrac{3\sqrt[3]{3150}}{150}\div3 = \dfrac{\sqrt[3]{3150}}{50}$

$5\cdot(2\cdot3\cdot5)$

2's two → need 1 more
3's one → need 2 more
5's one → need 2 more

9. $\dfrac{(3-\sqrt{2})(5-\sqrt{3})}{(5+\sqrt{3})(5-\sqrt{3})} = \dfrac{15-5\sqrt{2}-3\sqrt{3}+\sqrt{6}}{25-3} = \dfrac{15-5\sqrt{2}-3\sqrt{3}+\sqrt{6}}{22}$

we multiplied by the binomial conjugate

difference between two squares

$5^2-(\sqrt{3})^2$

PRACTICE: 251

1. $\sqrt{9}$ (3) 2. $\sqrt[3]{8}$ (2) 3. $\sqrt{8}$ ($2\sqrt{2}$) 4. $\sqrt[3]{16}$ ($2\sqrt[3]{2}$)

5. $5\sqrt{15} \times 4\sqrt{10}$ ($100\sqrt{6}$)

6. $7\sqrt[3]{75} \times 6\sqrt[3]{90}$ ($630\sqrt[3]{2}$)

7. $\frac{\sqrt{48}}{\sqrt{3}}$ (4) 8. $\frac{7}{\sqrt{5}}$ ($\frac{7\sqrt{5}}{5}$)

9. $\frac{24\sqrt{21}}{36\sqrt{14}}$ ($\frac{\sqrt{6}}{3}$)

10. $\frac{5\sqrt[3]{11}}{7\sqrt[3]{490}}$ ($\frac{\sqrt[3]{7700}}{98}$)

11. $\frac{5+\sqrt{3}}{7+\sqrt{5}}$ ($\frac{35+7\sqrt{3}-5\sqrt{5}-\sqrt{15}}{44}$)

12. $\frac{2-\sqrt{5}}{5+\sqrt{11}}$ ($\frac{10-5\sqrt{5}-2\sqrt{11}+\sqrt{55}}{14}$)

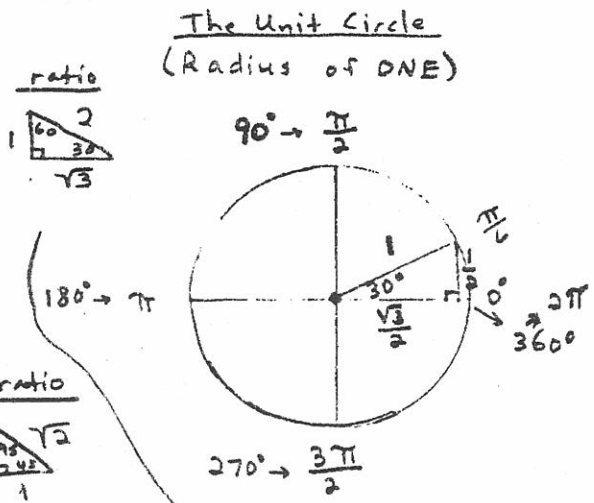
FORMULAS → because of similarity see p. 57

30° - 60° - 90° Triangle Rules

1. Hypotenuse = Short Leg × 2
2. Long Leg = Short Leg × $\sqrt{3}$

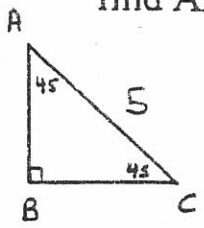
45° - 45° - 90° Triangle Rules

1. Legs are Equal
2. Hypotenuse = Leg × $\sqrt{2}$



EXAMPLES:

1. If AC is 5, then find AB and BC.



(solve first)

legs are =

$BC = AB$

$BC = \frac{5\sqrt{2}}{2}$

hyp = leg · $\sqrt{2}$

$\frac{5}{\sqrt{2}} = \frac{AB \cdot \sqrt{2}}{\sqrt{2}}$

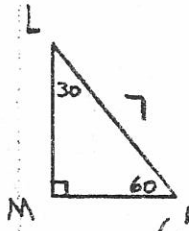
$AB = \frac{5}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{5\sqrt{2}}{2}$

dimensional analysis

$\frac{hyp}{1} \cdot \frac{leg}{hyp} = leg$

$\frac{5}{1} \cdot \frac{1}{\sqrt{2}} = \frac{5}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{5\sqrt{2}}{2}$

2. If LN is 7, then find LM and MN.



(solve first)

hyp = sl · 2

$\frac{7}{2} = \frac{MN \cdot 2}{2}$

$\frac{7}{2} = MN$

ll = sl · $\sqrt{3}$

$LM = \frac{7\sqrt{3}}{2}$

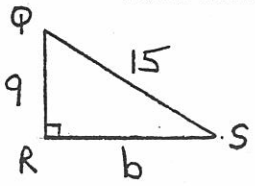
dimensional analysis

$\frac{hyp}{1} \cdot \frac{sl}{hyp} = sl$

$\frac{7}{1} \cdot \frac{1}{2} = \frac{7}{2}$

→ simplify see p. 248

3. If QR is 9 and QS is 15, then find RS.



$a = 9$
$b = b$
$c = 15$

$a^2 + b^2 = c^2$

$9^2 + b^2 = 15^2$

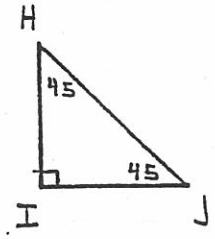
$81 + b^2 = 225$

$-81 \quad -81$

$\sqrt{b^2} = \sqrt{144}$

$b = 12$ or -12

4. If HJ is $13\sqrt{5}$ then find HI and IJ. 253



legs are =

$HI = IJ$

$HI = \frac{13\sqrt{10}}{2}$

hyp = l · √2

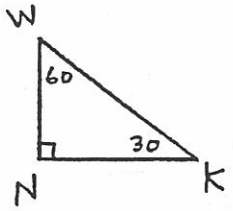
$\frac{13\sqrt{5}}{\sqrt{2}} = \frac{IJ\sqrt{2}}{\sqrt{2}}$

$IJ = \frac{13\sqrt{5} \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}}$

$IJ = \frac{13\sqrt{10}}{2}$

PRACTICE:

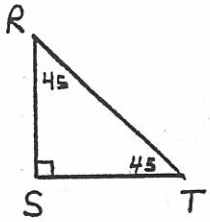
1. If WK is 9, then find WN and NK.



$WN = \frac{9}{2}$

$NK = \frac{9\sqrt{3}}{2}$

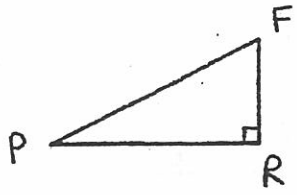
2. If RT is 11, then find RS and ST.



$RS = \frac{11\sqrt{2}}{2}$

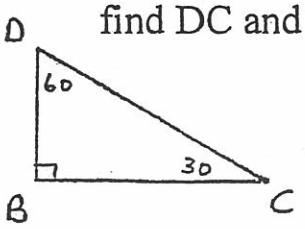
$ST = \frac{11\sqrt{2}}{2}$

3. If PF is 26 and FR is 10, then find RP.



$RP = 24$

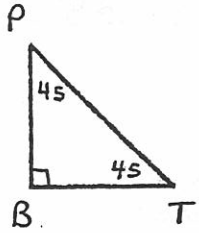
4. If DB is $9\sqrt{7}$, then find DC and CB.



$DC = 18\sqrt{7}$

$CB = 9\sqrt{21}$

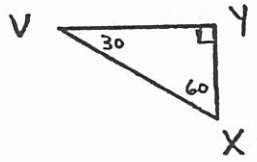
5. If PT is $5\sqrt{3}$, then find PB and BT.



$PB = \frac{5\sqrt{6}}{2}$

$BT = \frac{5\sqrt{6}}{2}$

6. If VX is 3, then find XY and VY.



$XY = \frac{3}{2}$

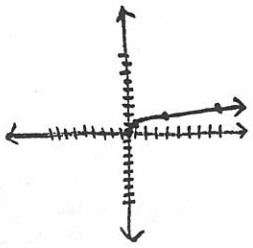
$VY = \frac{3\sqrt{3}}{2}$

1. $Y = \sqrt{X}$ or $y = X^{\frac{1}{2}}$

X	$\sqrt{X}$	y
-1	$\sqrt{-1}$	i (not real)
0	$\sqrt{0}$	0
1	$\sqrt{1}$	1
4	$\sqrt{4}$	2
9	$\sqrt{9}$	3

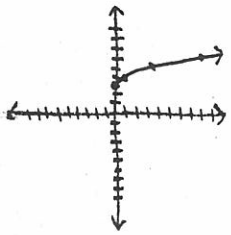
1 < 0, 3 < 1, 5 < 1

$\frac{\Delta y}{\Delta x}$ $\frac{1}{1}$ $\frac{1}{3}$ $\frac{1}{5}$ (not constant but Δ in Δx part is +2) power reg



2. $Y = \sqrt{X} + 3$ or $y = X^{\frac{1}{2}} + 3$ (shifted up 3)

X	$\sqrt{X}+3$	y
0	$\sqrt{0}+3$	3
1	$\sqrt{1}+3$	4
4	$\sqrt{4}+3$	5
9	$\sqrt{9}+3$	6

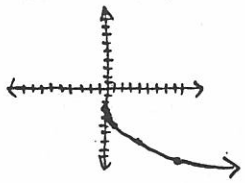


3. $Y = -2\sqrt{X} - 3$ → shifts down 3, arm folds out, reflects across the X-axis

X	$-2\sqrt{X}-3$	y
0	$-2\sqrt{0}-3$	-3
1	$-2\sqrt{1}-3$	-5
4	$-2\sqrt{4}-3$	-7
9	$-2\sqrt{9}-3$	-9

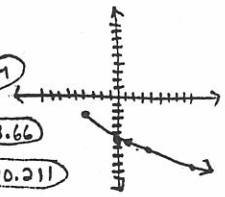
-2 -2 -2

$\frac{\Delta y}{\Delta x}$ $\frac{-2}{1}$ $\frac{-2}{3}$ $\frac{-2}{5}$ (+2, +2) (Δ in Δx part is +2)



4. $Y = -2\sqrt{X+4} - 3$ → reflects across the x-axis, arm folds out, shifts left 4, shifts down 3

X	$-2\sqrt{x+4}-3$	y
-4	$-2\sqrt{-4+4}-3$	-3
0	$-2\sqrt{0+4}-3$	-7
1	$-2\sqrt{1+4}-3$	$-2\sqrt{5}-3$ (-7.47)
4	$-2\sqrt{4+4}-3$	$-4\sqrt{2}-3$ (-8.66)
9	$-2\sqrt{9+4}-3$	$-2\sqrt{13}-3$ (-10.211)



5. X 1 2 3 4

Y 3 6 9 12

$\frac{\Delta y}{\Delta x}$ $\frac{3}{1}$ $\frac{3}{1}$ $\frac{3}{1}$

(constant so linear) linear reg

point (1, 3) slope 3

$y = mx + b$

$3 = 1 \cdot 3 + b$

$3 = 3 + b$

$-3 \quad -3$

$0 = b$

$y = 3x + 0$

$y = 3x$

6. X 1 2 3 4

Y 4 7 12 19

$\frac{\Delta y}{\Delta x}$ $\frac{3}{1}$ $\frac{5}{1}$ $\frac{7}{1}$

+2 +2

(Δ in Δy is +2 so of form $y = x^2$) quad reg

test

$1^2 = 1 + 3 = 4$

$2^2 = 4 + 3 = 7$

$3^2 = 9 + 3 = 12$

$y = x^2 + 3$

could do

X	so far (x^2)	need (y)
1	$1^2 = 1$	4
2	$2^2 = 4$	7
3	$3^2 = 9$	12
4	$4^2 = 16$	19

→ stat edit, stat calc, linear reg

7. X 1 2 3 4

Y -1 5 15 29

$\frac{\Delta y}{\Delta x}$ $\frac{6}{1}$ $\frac{10}{1}$ $\frac{14}{1}$

+4 +4

(Δ in Δy is +4 so of form $y = x^2$) quad reg

test

$2 \cdot 1^2 = 2 - 3 = -1$

$2 \cdot 2^2 = 8 - 3 = 5$

$2 \cdot 3^2 = 18 - 3 = 15$

$y = 2x^2 - 3$

could do

X	so far (x^2)	need (y)
1	$1^2 = 1$	-1
2	$2^2 = 4$	5
3	$3^2 = 9$	15

→ stat edit, stat calc, linear reg

8. X 1 4 9 16

Y 4 5 6 7

$\frac{\Delta y}{\Delta x}$ $\frac{1}{3}$ $\frac{1}{5}$ $\frac{1}{7}$

(Δ in Δx is +2 so of form $y = \sqrt{x}$) power reg

test

$\sqrt{1} = 1 + 3 = 4$

$\sqrt{4} = 2 + 3 = 5$

X	so far $\sqrt{x}$	need (y)
1	1	4
4	2	5
9	3	6
16	4	7

$y = \sqrt{x} + 3$ or $y = x^{1/2} + 3$

9. X 1 4 9 16

Y -1 1 3 5

$\frac{\Delta y}{\Delta x}$ $\frac{2}{3}$ $\frac{2}{5}$ $\frac{2}{7}$

(Δ in Δx is +2 so of form $y = \sqrt{x}$; Δy is 2 so of form $y = 2\sqrt{x}$) power reg

test

$2\sqrt{1} = 2 - 3 = -1$

$2\sqrt{4} = 4 - 3 = 1$

$2\sqrt{9} = 6 - 3 = 3$

$y = 2\sqrt{x} - 3$

X	so far $\sqrt{x}$	need (y)
1	1	-1
4	2	1
9	3	3
16	4	5

stat edit, stat calc, linear reg

10. X 1 2 3 4

Y 7 9 13 21

$\frac{\Delta y}{\Delta x}$ $\frac{2}{1}$ $\frac{4}{1}$ $\frac{8}{1}$

(Δ in Δy is ·2 so of the form $y = 2^x$) exp reg

test

$2^1 = 2 + 5 = 7$

$2^2 = 4 + 5 = 9$

$2^3 = 8 + 5 = 13$

$y = 2^x + 5$

X	so far 2^x	need (y)
1	2	7
2	4	9
3	8	13
4	16	21

stat edit, stat calc, linear reg

11. X 1 2 3 4

Y -1 5 23 77

$\frac{\Delta y}{\Delta x}$ $\frac{6}{1}$ $\frac{18}{1}$ $\frac{54}{1}$

(Δ in Δy is ·3 so of the form $y = 3^x$) exp reg

test

$3^1 = 3 - 4 = -1$

$3^2 = 9 - 4 = 5$

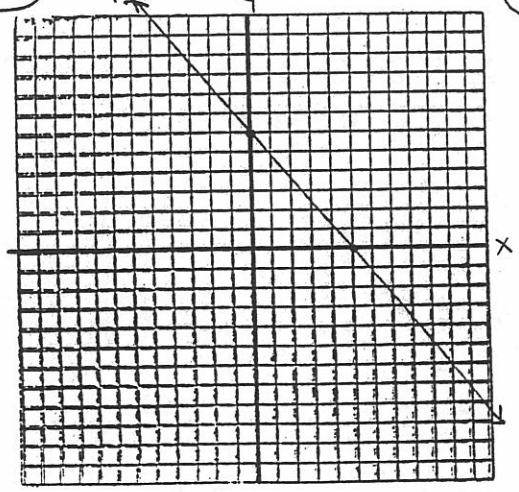
$3^3 = 27 - 4 = 23$

$y = 3^x - 4$

X	so far 3^x	need (y)
1	3	-1
2	9	5
3	27	23
4	81	77

stat edit, stat calc, linear reg

12.



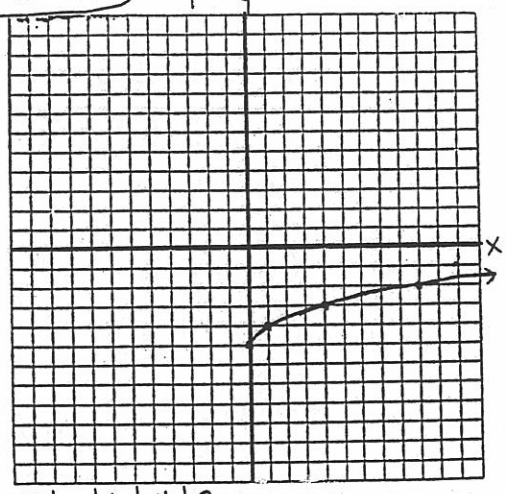
linear

y int: 6

slope: $\frac{\text{rise}}{\text{run}}$ $\frac{\text{up } 6}{\text{left } 5}$ $\frac{6}{-5}$

$y = \frac{-6}{5}x + 6$

13.



X	0	1	4	9
y	-5	-4	-3	-2

$y = \sqrt{x} - 5$

$\frac{\Delta y}{\Delta x}$ $\frac{1}{1}$ $\frac{1}{3}$ $\frac{1}{5}$ $y = \sqrt{x}$

Δ in Δx: +2 +2

X	so far $\sqrt{x}$	need (y)
0	0	-5
1	1	-4
4	2	-3
9	3	-2