CHAPTER 9
RADICALS

9.1 ADDING AND SUBTRACTING RADICALS

RULE 1: To add or subtract you have to have the same name. In Algebra we call them like terms. (Remember , every little piece of the names must be identical.) see p.1

EXAMPLE:

1. $3\sqrt{3}+5\sqrt{3} =$ $8\sqrt{3}$
2. $3\sqrt{3}+5\sqrt{7} =$ simplest form
3. $9\sqrt{6} - 4\sqrt{6} =$ $5\sqrt{6}$
4. $8\sqrt{11} - 3\sqrt{5} =$ simplest form
5. $7XY^2\sqrt{5}+4XY^2\sqrt{5} =$ $11XY^2\sqrt{5}$
6. $9XY^2\sqrt{5}+3XY\sqrt{5} =$ simplest form

PRACTICE

1. $5\sqrt{7}+8\sqrt{7} =$
2. $11\sqrt{13} - 5\sqrt{13} =$
3. $7\sqrt{5}+3\sqrt{5} =$
4. $13MN^3\sqrt{17}+4MN^3\sqrt{17} =$
5. $19XH\sqrt{5} - 11XH^2\sqrt{5} =$
6. $7CH\sqrt{3}+3CH\sqrt{3} =$
7. $3KP\sqrt{11}+7K^3P^2\sqrt{11}+5KP\sqrt{11}+8K^3P^2\sqrt{11} =$

9.2 MULTIPLYING AND DIVIDING RADICALS

RULE 2: To multiply or divide you do NOT have to have the same name. They do NOT have to be like terms. Every time you multiply or divide you get a new name. (Don't forget exponent , distributive , and sign rules.)

EXAMPLE: see p. 5

1. $4\sqrt{3} \times 5\sqrt{7} = 20\sqrt{21}$

2. $16\sqrt{15} \div 8\sqrt{3} = 2\sqrt{5}$

3. $7\sqrt{11} \times 4\sqrt{5} = 28\sqrt{55}$

4. $-24\sqrt{21} \div -6\sqrt{7} = 4\sqrt{3}$

5. $-5X^3M^5\sqrt{3} \times 7X^2M^2Q\sqrt{11} =$
$-35X^5M^7Q\sqrt{33}$

6. $\dfrac{28TR^5\sqrt{21}}{49T^3R^2\sqrt{7}} = \dfrac{4R^3\sqrt{3}}{7T^2}$

PRACTICE:

1. $-8\sqrt{13} \times -7\sqrt{5} =$

2. $6\sqrt{35} \div 3\sqrt{5} =$

3. $-7FH\sqrt{5} \times 9FH^3\sqrt{3} =$

4. $3Q\sqrt{13} \times 7D\sqrt{11} =$

5. $18X^5B^7\sqrt{55} \div 2X^3B\sqrt{5} =$

6. $5XY\sqrt{7}(-3X^2Y\sqrt{3} + -4X^3Y^3\sqrt{5}) + 8X^3Y^2\sqrt{21} =$

9.3 SIMPLIFYING RADICALS see p. 15 + p. 24

RULE 1: To simplify a radical, you should ask yourself these four questions, in this order, after every single step.

1. Will anything reduce? (#'s with #'s $\sqrt{\ }$ with $\sqrt{\ }$)
2. Will a radical break down? (have sets of primes)
3. Is there a fraction in the $\sqrt{\ }$?
4. Is there a $\sqrt{\ }$ in the denominator?

When you get a NO to all four questions then it must be in simplest form.

FIXING PROBLEM 1 (REDUCING) see p. 5

$$\frac{28\sqrt{48} \div 7\sqrt{2}}{35\sqrt{10} \div 7\sqrt{2}} = \frac{4\sqrt{24}}{5\sqrt{5}}$$

(still has problems)

$$\frac{48\sqrt{35} \div 16\sqrt{5}}{32\sqrt{55} \div 16\sqrt{5}} = \frac{3\sqrt{7}}{2\sqrt{11}}$$

(still has problems)

FIXING PROBLEM 2 (BREAKING DOWN RADICALS)

1. Do the prime factorization? (Primes -- 2,3,5,7,11,13,17,19, ect.)
2. Circle sets of 2 for $\sqrt{\ }$, 3 for $\sqrt[3]{\ }$, ect. see p. 15
3. Take one from each set outside.
4. Leave those not in a set inside.
5. Multiply the outsides. Multiply the insides.

$\sqrt{4720}$

$\sqrt{2\cdot2\cdot2\cdot2\cdot5\cdot59}$

$2\cdot2\sqrt{5\cdot59}$

$\boxed{4\sqrt{295}}$

$$\begin{array}{r|l} 2 & 4720 \\ 2 & 2360 \\ 2 & 1180 \\ 2 & 590 \\ 5 & 295 \\ & 59 \end{array}$$

$\sqrt[3]{4720}$

$\sqrt[3]{2\cdot2\cdot2\cdot2\cdot5\cdot59}$

$2\sqrt[3]{2\cdot5\cdot59}$

$\boxed{2\sqrt[3]{590}}$

$\sqrt{70}$

simplest form (no sets)

$$\begin{array}{r|l} 2 & 70 \\ 5 & 35 \\ & 7 \end{array}$$

FIXING PROBLEM 3 (A FRACTION IN THE $\sqrt{\ }$)

To fix this problem just split it apart.

$$\sqrt{\frac{2}{5}} = \frac{\sqrt{2}}{\sqrt{5}} \quad \left(\text{still has problems}\right) \qquad \sqrt{.3} = \sqrt{\frac{3}{10}} = \frac{\sqrt{3}}{\sqrt{10}}$$

Once it is split , we have the fourth problem. A $\sqrt{\ }$ in the denominator.

FIXING PROBLEM 4 (A $\sqrt{\ }$ IN THE DENOMINATOR)

RULE: To rationalize a denominator (get the $\sqrt{\ }$ off) , you multiply by $\sqrt{\_ \times \_ \times \_}$ or $\sqrt[3]{\_ \times \_ \times \_}$ ect. so that each prime will appear the index number of times.

(index $\rightarrow$ 2 for $\sqrt{\ }$ 3 for $\sqrt[3]{\ }$ ect.)

See page 15

REASON: $\underbrace{\sqrt{5} \times \sqrt{5}}_{\sqrt{5\cdot5}} = 5 \qquad \underbrace{\sqrt[3]{7} \times \sqrt[3]{7} \times \sqrt[3]{7}}_{\sqrt[3]{7\cdot7\cdot7}} = 7$

$$\frac{5\sqrt{7}\cdot\sqrt{5}}{\underbrace{3\sqrt{5}\cdot\sqrt{5}}_{\sqrt{5\cdot5}}} = \frac{\cancel{5}\sqrt{35}}{3\cdot\cancel{5}} = \frac{\sqrt{35}}{3}$$

$$\frac{4\sqrt[3]{11}}{9\sqrt[3]{3}}\cdot\frac{\sqrt[3]{3\cdot3}}{\underbrace{\sqrt[3]{3\cdot3}}_{\sqrt[3]{3\cdot3\cdot3}}} = \frac{4\sqrt[3]{99}}{9\cdot3} = \frac{4\sqrt[3]{99}}{27}$$

EXAMPLE:

1. $$\frac{55\sqrt{15} \div 11\sqrt{5}}{66\sqrt{35} \div 11\sqrt{5}} = \frac{5\sqrt{3}\cdot\sqrt{7}}{\underbrace{6\sqrt{7}\cdot\sqrt{7}}_{6\cdot7}} = \frac{5\sqrt{21}}{42}$$

2. $4\sqrt{6} \times 7\sqrt{30} = 28\sqrt{180} = 28\sqrt{2\cdot2\cdot3\cdot3\cdot5}$

$28\cdot2\cdot3\sqrt{5}$

$168\sqrt{5}$

$$\begin{array}{r|l} 2 & 180 \\ 2 & 90 \\ 3 & 45 \\ 3 & 15 \\ & 5 \end{array}$$

3. $\sqrt{48}$

$$\begin{array}{r|l} 2 & 48 \\ 2 & 24 \\ 2 & 12 \\ 2 & 6 \\ & 3 \end{array}$$

$\sqrt{(2\cdot2)(2\cdot2)3}$

$2\cdot2\sqrt{3}$

$4\sqrt{3}$

4. $\dfrac{\sqrt{24}\div\sqrt{6}}{\sqrt{6}\div\sqrt{6}}$

$\sqrt{4} = 2$

5. $\dfrac{2}{\sqrt{2}}\cdot\dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{\cancel{2}\sqrt{2}}{\cancel{2}}$

$\sqrt{2}$

6. $\dfrac{42\sqrt[3]{50}\div 6\sqrt[3]{10}}{36\sqrt[3]{30}\div 6\sqrt[3]{10}} = \dfrac{7\sqrt[3]{5}\cdot\sqrt[3]{3\cdot3}}{6\sqrt[3]{3}\cdot\sqrt[3]{3\cdot3}} = \dfrac{7\sqrt[3]{45}}{18}$

$\hookrightarrow 6\cdot 3$

for how to multiply the numerators please see p. 204

we multiplied by the binomial conjugate see page 200

7. $\dfrac{(3+\sqrt{5})}{(2+\sqrt{7})}\dfrac{(2-\sqrt{7})}{(2-\sqrt{7})} = \dfrac{6+2\sqrt{5}-3\sqrt{7}-\sqrt{35}}{4-7} = \dfrac{6+2\sqrt{5}-3\sqrt{7}-\sqrt{35}}{-3}$

difference between two squares

$2^2-(\sqrt{7})^2$

see page 204

$$\begin{array}{r|l} 2 & 60 \\ 2 & 30 \\ 3 & 15 \\ & 5 \end{array}$$

8. $\dfrac{3\sqrt[3]{7}\cdot\sqrt[3]{2\cdot3\cdot3\cdot5\cdot5}}{5\sqrt[3]{60}\cdot\sqrt[3]{2\cdot3\cdot3\cdot5\cdot5}} = \dfrac{3\sqrt[3]{3150}\div3}{150\div3} = \dfrac{\sqrt[3]{3150}}{50}$

$\hookrightarrow 5\cdot(2\cdot3\cdot5)$

2's two → need 1 more
3's one → need 2 more
5's one → need 2 more

we multiplied by the binomial conjugate

9. $\dfrac{(3-\sqrt{2})}{(5+\sqrt{3})}\dfrac{(5-\sqrt{3})}{(5-\sqrt{3})} = \dfrac{15-5\sqrt{2}-3\sqrt{3}+\sqrt{6}}{25-3} = \dfrac{15-5\sqrt{2}-3\sqrt{3}+\sqrt{6}}{22}$

difference between two squares

$5^2-(\sqrt{3})^2$

PRACTICE:

1. $\sqrt{9}$ 2. $\sqrt[3]{8}$ 3. $\sqrt{8}$ 4. $\sqrt[3]{16}$

5. $5\sqrt{15} \times 4\sqrt{10}$

6. $7\sqrt[3]{75} \times 6\sqrt[3]{90}$

7. $\dfrac{\sqrt{48}}{\sqrt{3}}$ 8. $\dfrac{7}{\sqrt{5}}$

9. $\dfrac{24\sqrt{21}}{36\sqrt{14}}$

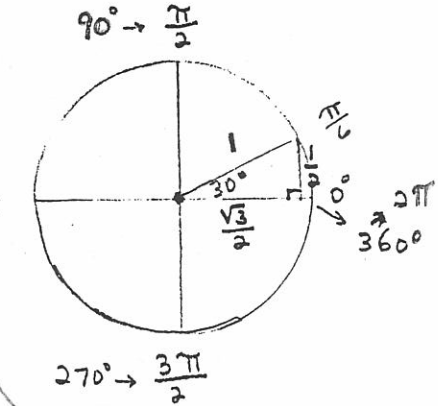
10. $\dfrac{5\sqrt[3]{11}}{7\sqrt[3]{490}}$

11. $\dfrac{5+\sqrt{3}}{7+\sqrt{5}}$

12. $\dfrac{2-\sqrt{5}}{5+\sqrt{11}}$

FORMULAS because of similarity see p. 57

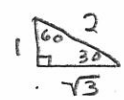
The Unit Circle
(Radius of ONE)



30° - 60° - 90° Triangle Rules

1. Hypotenuse = Short Leg × 2
2. Long Leg = Short Leg × $\sqrt{3}$

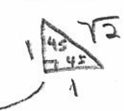
ratio



45° - 45° - 90° Triangle Rules

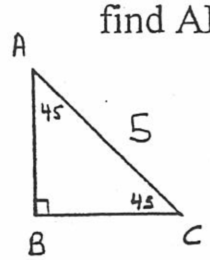
1. Legs are Equal
2. Hypotenuse = Leg × $\sqrt{2}$

ratio

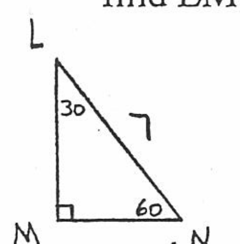


EXAMPLES:

1. If AC is 5 , then find AB and BC.



2. If LN is 7 , then find LM and MN.



(solve first)

legs are =

BC = AB

BC = $\frac{5\sqrt{2}}{2}$

hyp = leg · $\sqrt{2}$

$\frac{5}{\sqrt{2}} = \frac{AB \cdot \sqrt{2}}{\sqrt{2}}$

AB = $\frac{5}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{5\sqrt{2}}{2}$

simplify see p. 248

dimensional analysis

$\frac{hyp}{1} \cdot \frac{leg}{hyp} = leg$

$\frac{5}{1} \cdot \frac{1}{\sqrt{2}} = \frac{5}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{5\sqrt{2}}{2}$

(solve first)

hyp = sl · 2

$\frac{7}{2} = \frac{MN \cdot 2}{2}$

$\frac{7}{2}$ = MN

ll = sl · $\sqrt{3}$

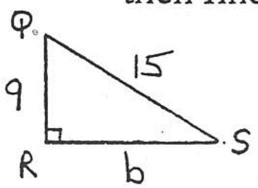
LM = $\frac{7\sqrt{3}}{2}$

dimensional analysis

$\frac{hyp}{1} \cdot \frac{sl}{hyp} = sl$

$\frac{7}{1} \cdot \frac{1}{\ }$ = $\frac{7}{\ }$

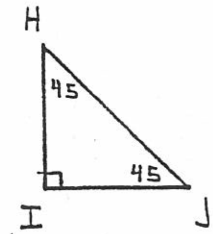
3. If QR is 9 and QS is 15 , then find RS.



$a = 9$
$b = b$
$c = 15$

$a^2 + b^2 = c^2$
$9^2 + b^2 = 15^2$
$81 + b^2 = 225$
$-81 \qquad -81$
$\sqrt{b^2} = \sqrt{144}$
$b = 12$ or $\cancel{-12}$

4. If HJ is $13\sqrt{5}$ then find HI and IJ.

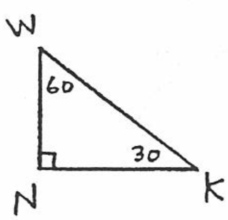


legs are =
HI = IJ
$HI = \frac{13\sqrt{10}}{2}$

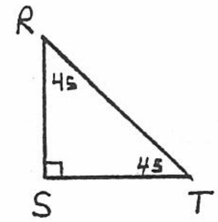
$hyp = l \cdot \sqrt{2}$
$\frac{13\sqrt{5}}{\sqrt{2}} = \frac{IJ\sqrt{2}}{\sqrt{2}}$
$IJ = \frac{13\sqrt{5} \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}}$
$IJ = \frac{13\sqrt{10}}{2}$

PRACTICE:

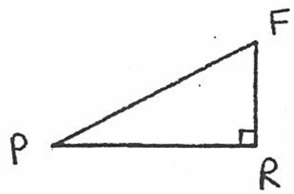
1. If WK is 9 , then find WN and NK.



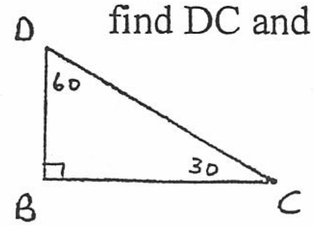
2. If RT is 11 , then find RS and ST.



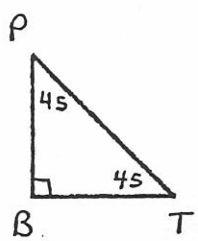
3. If PF is 26 and FR is 10 , then find RP.



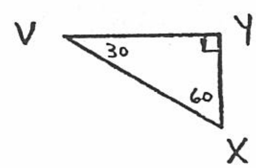
4. If DB is $9\sqrt{7}$, then find DC and CB. 254



5. If PT is $5\sqrt{3}$, then find PB and BT.



6. If VX is 3 , then find XY and VY.

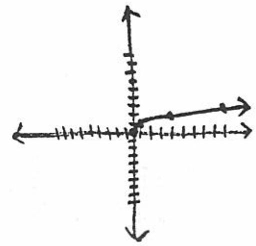


see p. 13

1. $Y = \sqrt{X}$ or $y = x^{\frac{1}{2}}$

X	$\sqrt{X}$	y
-1	$\sqrt{-1}$	i (not real)
0	$\sqrt{0}$	0
1	$\sqrt{1}$	1
4	$\sqrt{4}$	2
9	$\sqrt{9}$	3

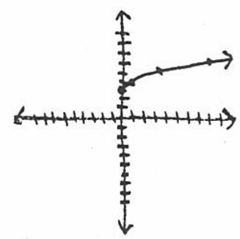
$\frac{\Delta y}{\Delta x}$ $\frac{1}{1}$ $\frac{1}{3}$ $\frac{1}{5}$ (not constant but Δ in Δx part is +2) power reg



2. $Y = \sqrt{X} + 3$ or $y = x^{1/2} + 3$

shifted up 3

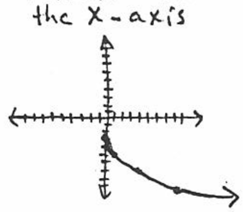
X	$\sqrt{X} + 3$	y
0	$\sqrt{0} + 3$	3
1	$\sqrt{1} + 3$	4
4	$\sqrt{4} + 3$	5
9	$\sqrt{9} + 3$	6



3. $Y = -2\sqrt{X} - 3$

shifts down 3
arm folds out
reflects across the x-axis

X	$-2\sqrt{x} - 3$	y
0	$-2\sqrt{0} - 3$	-3
1	$-2\sqrt{1} - 3$	-5
4	$-2\sqrt{4} - 3$	-7
9	$-2\sqrt{9} - 3$	-9

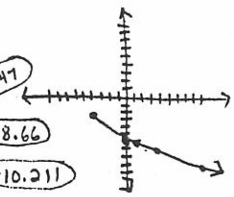


$\frac{\Delta y}{\Delta x}$ $\frac{-2}{1}$ $\frac{-2}{3}$ $\frac{-2}{5}$ (+2, +2) (Δ in Δx part is +2)

4. $Y = -2\sqrt{X + 4} - 3$

reflects across the x-axis
arm folds out
shifts left 4
shifts down 3

X	$-2\sqrt{x+4} - 3$	y
-4	$-2\sqrt{-4+4} - 3$	-3
0	$-2\sqrt{0+4} - 3$	-7
1	$-2\sqrt{1+4} - 3$	$-2\sqrt{5} - 3$ (-7.47)
4	$-2\sqrt{4+4} - 3$	$-4\sqrt{2} - 3$ (-8.66)
9	$-2\sqrt{9+4} - 3$	$-2\sqrt{13} - 3$ (-10.211)



see regression rules at 1st of book

5. X 1 2 3 4

Y 3 6 9 12

$\frac{\Delta y}{\Delta x}$ $\frac{3}{1}$ $\frac{3}{1}$ $\frac{3}{1}$

(constant so linear) linear reg

point (1, 3) slope 3

$y = mx + b$

$3 = 1 \cdot 3 + b$

$3 = 3 + b$

$-3 \quad -3$

$0 = b$

$y = 3x + 0$

$\boxed{y = 3x}$

6. X 1 2 3 4

Y 4 7 12 19

$\frac{\Delta y}{\Delta x}$ $\frac{3}{1}$ $\frac{5}{1}$ $\frac{7}{1}$

+2 +2

(Δ in Δy is +2 so of form $y = x^2$) quad reg

test

$1^2 = 1 + 3 = 4$

$2^2 = 4 + 3 = 7$

$3^2 = 9 + 3 = 12$

$\boxed{y = x^2 + 3}$

could do

X	so far (x^2)	need (y)
1	$1^2 = 1$	4
2	$2^2 = 4$	7
3	$3^2 = 9$	12
4	$4^2 = 16$	19

→ stat edit, stat calc, linear reg

7. X 1 2 3 4

Y -1 5 15 29

$\frac{\Delta y}{\Delta x}$ $\frac{6}{1}$ $\frac{10}{1}$ $\frac{14}{1}$

+4 +4

(Δ in Δy is +4 so of form $y = x^2$) quad reg

test

$2 \cdot 1^2 = 2 - 3 = -1$

$2 \cdot 2^2 = 8 - 3 = 5$

$2 \cdot 3^2 = 18 - 3 = 15$

$\boxed{y = 2x^2 - 3}$

could do

X	so far (x^2)	need (y)
1	$1^2 = 1$	-1
2	$2^2 = 4$	5
3	$3^2 = 9$	15
4	$4^2 = 16$	29

→ stat edit, stat calc, linear reg

8. X 1 4 9 16

Y 4 5 6 7

$\frac{\Delta y}{\Delta x}$ $\frac{1}{3}$ $\frac{1}{5}$ $\frac{1}{7}$

(Δ in Δx is +2 so of form $y=\sqrt{x}$) power reg

test

$\sqrt{1} = 1 + 3 = 4$

$\sqrt{4} = 2 + 3 = 5$

X	so far $\sqrt{x}$	need (y)
1	1	4
4	2	5
9	3	6
16	4	7

$y=\sqrt{x}+3$ or $y = x^{1/2} + 3$

9. X 1 4 9 16

Y -1 1 3 5

$\frac{\Delta y}{\Delta x}$ $\frac{2}{3}$ $\frac{2}{5}$ $\frac{2}{7}$

(Δ in Δx is +2 so of form $y=\sqrt{x}$, Δy is 2 so of form $y = 2\sqrt{x}$) power reg

test

$2\sqrt{1} = 2 - 3 = -1$

$2\sqrt{4} = 4 - 3 = 1$

$2\sqrt{9} = 6 - 3 = 3$

X	so far $2\sqrt{x}$	need (y)
1	1	-1
4	2	1
9	3	3
16	4	5

stat edit
stat calc
linear reg

$y = 2\sqrt{x} - 3$

10. X 1 2 3 4

Y 7 9 13 21

$\frac{\Delta y}{\Delta x}$ $\frac{2}{1}$ $\frac{4}{1}$ $\frac{8}{1}$

(Δ in Δy is $\cdot 2$ so of the form $y = 2^x$) exp reg

test

$2^1 = 2 + 5 = 7$

$2^2 = 4 + 5 = 9$

$2^3 = 8 + 5 = 13$

X	so far 2^x	need (y)
1	2	7
2	4	9
3	8	13
4	16	21

stat edit
stat calc
linear reg

$y = 2^x + 5$

11. X 1 2 3 4

Y -1 5 23 77

$\frac{\Delta y}{\Delta x}$ $\frac{6}{1}$ $\frac{18}{1}$ $\frac{54}{1}$

(Δ in Δy is $\cdot 3$ so of the form $y = 3^x$) exp reg

test

$3^1 = 3 - 4 = -1$

$3^2 = 9 - 4 = 5$

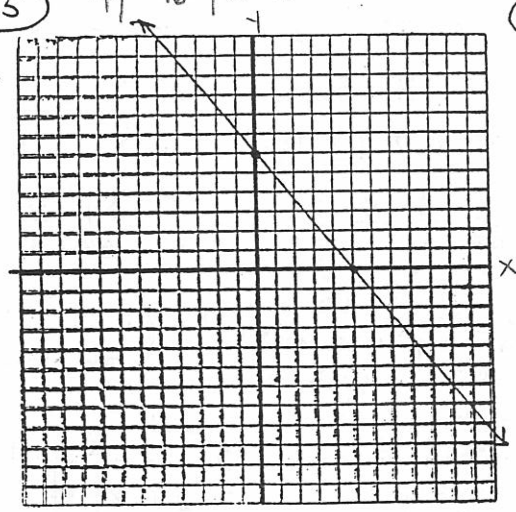
$3^3 = 27 - 4 = 23$

X	so far 3^x	need (y)
1	3	-1
2	9	5
3	27	23
4	81	77

stat edit
stat calc
linear reg

$y = 3^x - 4$

12.



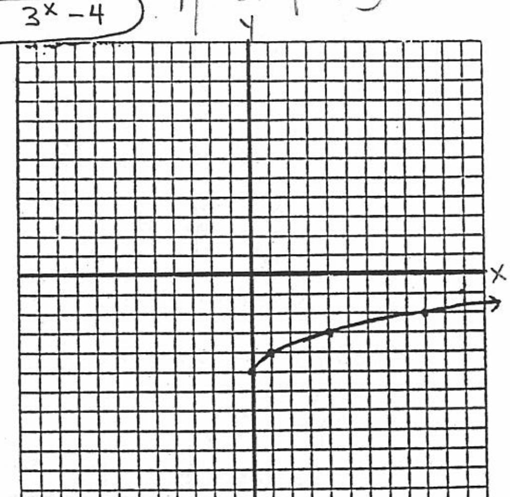
linear

y int : 6

slope: $\frac{\text{rise}}{\text{run}}$ $\frac{\text{up } 6}{\text{left } 5}$ $\frac{6}{-5}$

$y = \frac{-6}{5}x + 6$

13.



X	0	1	4	9
Y	-5	-4	-3	-2

$y = \sqrt{x} - 5$

$\frac{\Delta y}{\Delta x}$ $\frac{1}{1}$ $\frac{1}{3}$ $\frac{1}{5}$ } $y=\sqrt{x}$

Δ in Δx +2 +2

X	so far $\sqrt{x}$	need (y)
0	0	-5
1	1	-4
4	2	-3
9	3	-2

PRACTICE: (Supply the missing table , graph , or equation.)

1. $Y = -3X + 4$

2. $Y = -3\sqrt{X + 4} + 2$

3.

X	1	2	3	4
Y	2	0	-2	-4

4.

X	1	2	3	4
Y	-8	1	16	37

5.

X	1	4	9	16
Y	1	3	5	7

6.

X	1	2	3	4
Y	-16	-4	44	236